Emittance in Tevatron

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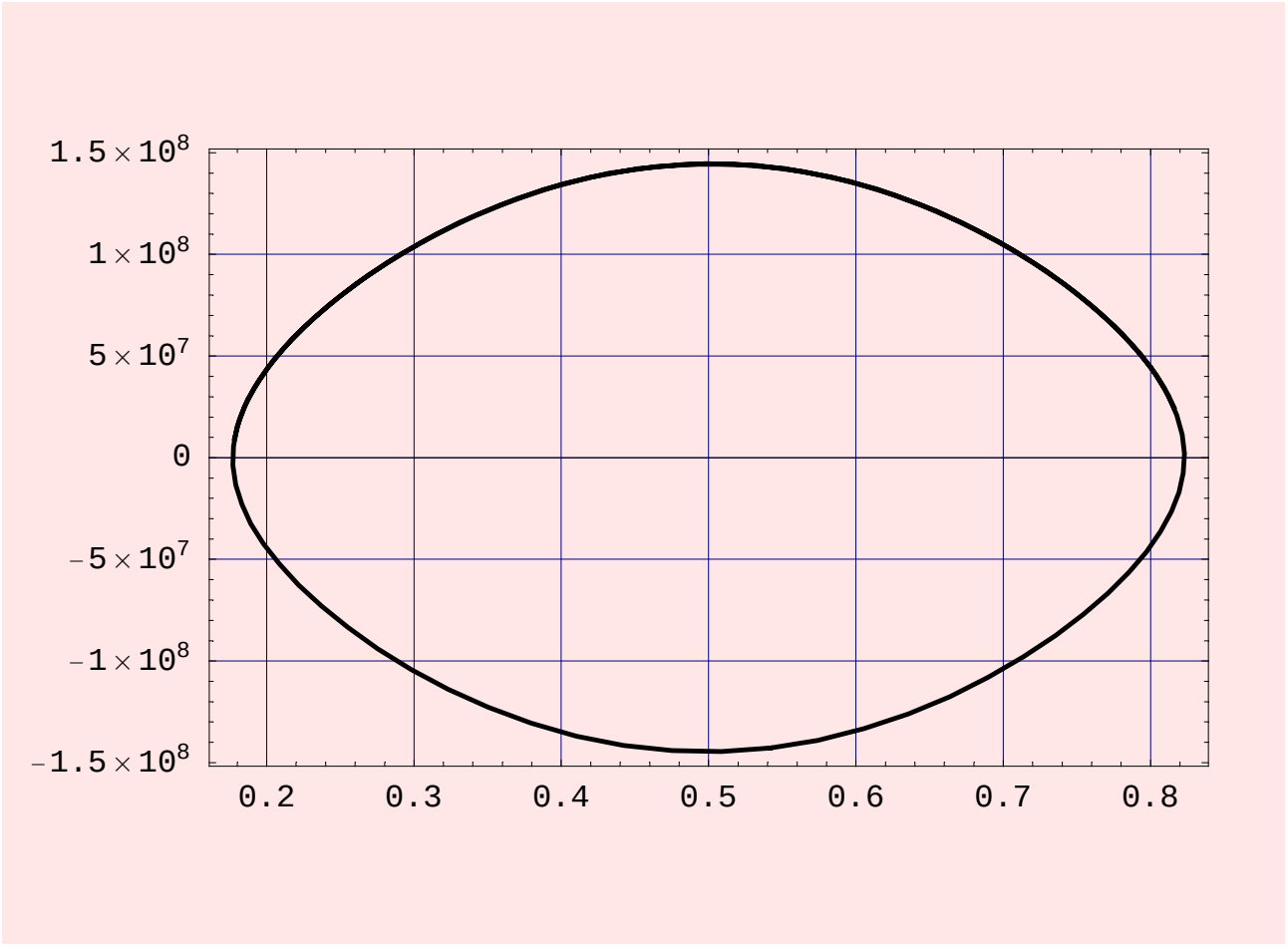
THINGS THAT AFFECT THE EMITTANCE CALCULATIONS:

1. THE DISCRETE NATURE OF THE ACCELERATION PROCESS.

2. THE PHASE SPACE DISTRIBUTION. WE ONLY MEASURE THE DISTRIBUTION ALONG THE T OR Z AXIS. WE WOULD LIKE TO KNOW FROM THIS THE EMITTANCE AND THE MOMENT IN THE P DIRECTION.

TURN BY TURN ACCELERATION GIVES A SLIGHTLY DIFFERENT ANSWER FOR THE TRAJECTORIES IN THE E,T SPACE THAN SETTING H=CONSTANT.

$$E^2 + \frac{2 E_S V_{RF}}{2 \pi \eta h} \text{Cos}[\phi] = \text{constant}$$



THE RATIO IS INDEPENDENT OF THE SIZE OF THE ORBIT AND OF THE MACHINE ENERGY. THIS MEANS THAT THE MOMENTUM SPREAD DERIVED USING THE HAMALTONIAN SHOULD BE DECREASED BY THE FACTOR 1.055

Z0	continuous	discrete	Ratio
0.0001	1.79665×10^{8}	1.70297×10^{8}	1.05501
0.2001	1.78553×10^{8}	1.69283×10^{8}	1.05476
0.4001	1.75236×10^{8}	1.66163×10^{8}	1.0546
0.6001	1.69753×10^{8}	1.60964×10^{8}	1.0546
0.8001	1.62172×10^{8}	1.53776×10^{8}	1.0546
1.0001	1.52588×10^{8}	1.44688×10^{8}	1.0546
1.2001	1.41117×10^{8}	1.33811×10^{8}	1.05461
1.4001	1.27903×10^{8}	1.21282×10^{8}	1.0546
1.6001	1.13109×10^{8}	1.07254×10^{8}	1.05459
1.8001	9.69167×10^{7}	9.18989×10^{7}	1.0546
2.0001	7.95269×10^{7}	7.54099×10^{7}	1.0546
2.2001	6.11544×10^{7}	5.79887×10^{7}	1.05459
2.4001	4.20261×10^{7}	3.98507×10^{7}	1.05459
2.6001	2.23786×10^{7}	2.12202×10^{7}	1.05459
2.8001	2.45449×10^{6}	2.32737×10^{6}	1.05462

2. THE PHASE SPACE DISTRIBUTION AND THE CHURCH ANSATZ.

SOME FACTS:

A. IF WE HAVE AN EQUILIBRIUM DISTRIBUTION, THEN BY LIOUVILLE'S THEOREM THE PHASE SPACE DENSITY IS CONSTANT ALONG A PHASE TRAJECTORY ...IE THE ELLIPSE LIKE CURVES GIVEN BY H=constant. THUS, THERE IS ONLY ONE VARIABLE THAT THE DENSITY DEPENDS ON. THIS VARIABLE IS TAKEN AS THE ACTION.

ACTION=$\oint$P dZ = AREA OF THE PHASE ELLIPSE.

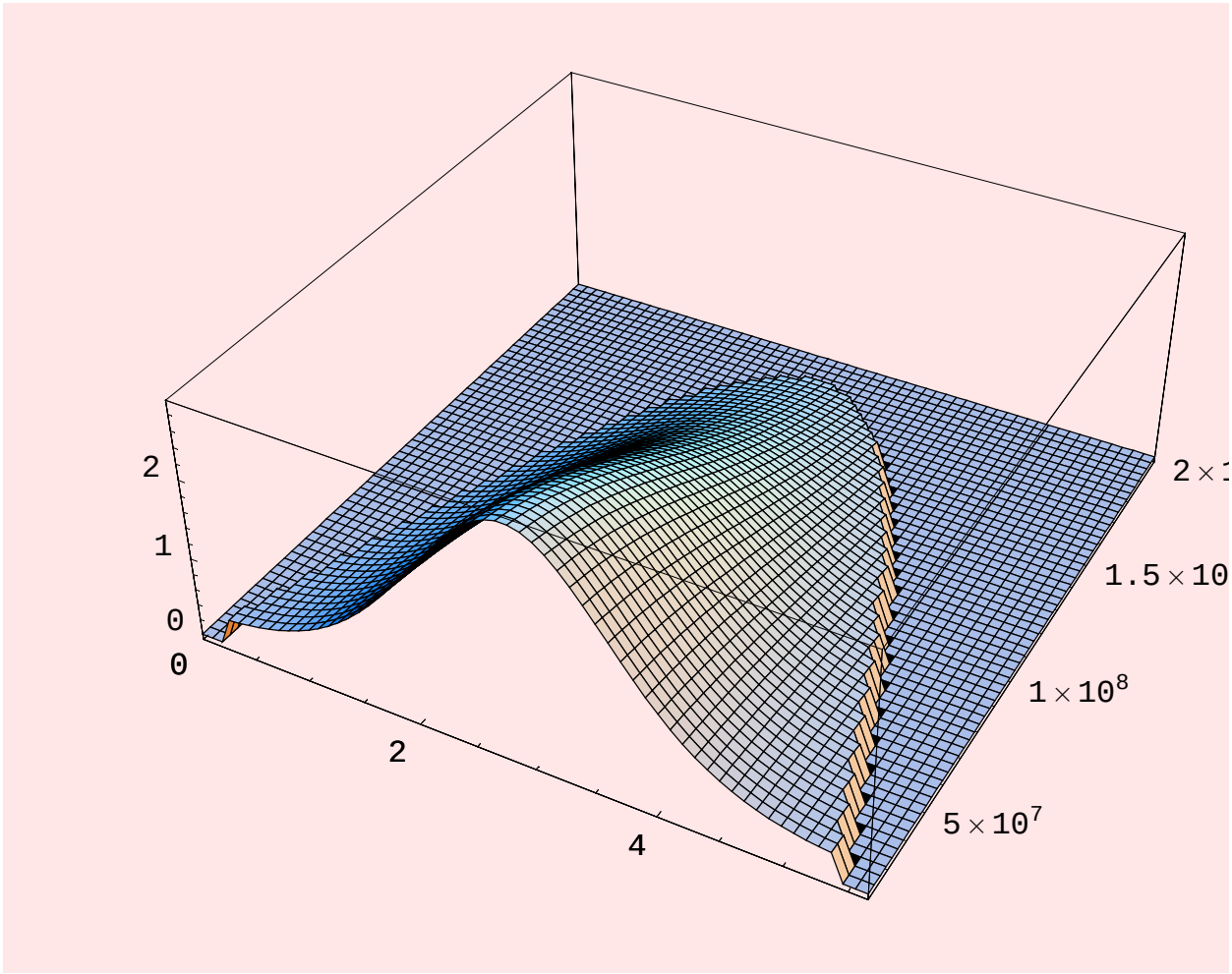
ρ=F[ACTION] IS THE MOST GENERAL FORM FOR THE DENSITY.

CHURCH ASSUMPTION: $\rho=\rho$[E] ρ[Z]. THIS LEADS TO A UNIQUE SOLUTION FOR ρ:

$$\rho = C * e^{\frac{E^2}{2\sigma_E^2}} * e^{\frac{\mathrm{Cos}[\phi]}{\sigma_\phi^2}} \quad \text{and} \quad \sigma_\phi = \sqrt{\frac{2\pi h\eta}{E_s V_{RF}}}\ \sigma_E$$

DOES THIS SATISFY THE STATEMENT THAT ρ[ACTION]? YES. THE COMBINATION OF THE TWO TERMS IN THE EXPONENTIAL WHEN COMBINED IS JUST PROPORTIONAL TO H!

CHURCH USED THE ABOVE FORM WITH A MC TECHNIQUE TO RELATE Trms to Erms BY THOWING DISTRIBUTIONS WITH DIFFERENT σ_E AND CALCULATING Trms AND Erms. NOTE THAT THERE IS NO CUTOFF AT THE SEPARATRIX, SO THE DISTRIBUTION IS TRUNCATED AT THE EDGE. I HAVE DUPLICATED THIS ANALYTICALLY. A TYPICAL DISTRIBUTION LOOKS LIKE THE PLOT BELOW.



sigE, Church Input	rmsE	rmsT	rmsE/rmsT
0	0	0	
10	$1.\times 10^7$	0.334909	29.8588
30	$3.\times 10^7$	1.03321	29.0358
50	4.96907×10^7	1.85931	26.7254
70	6.46794×10^7	2.69044	24.0404
90	7.27374×10^7	3.21196	22.6458

THE FOLLOWING EQUATION FROM CHURCH GIVES Erms IN TERMS OF Trms. THE TABLE COMPARES MY ANALYTICAL CALCULATION OF Erms WITH THIS EQUATION. FOR SMALL Trms THE COEFFICIENT OF T SHOULD BE 29.85

```
fChurch[t_]:=32.66 t -3.527 t^2 +.1314 t^3
```

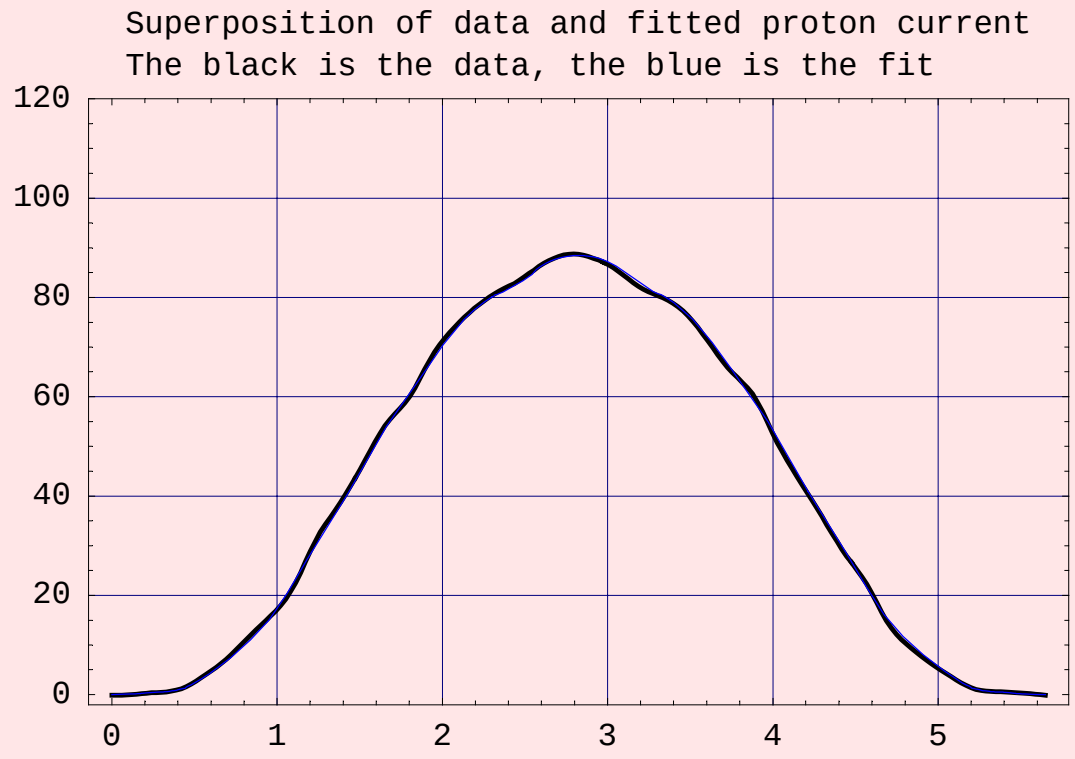
AVT calc	Church
0	0
10.	10.5475
30.	30.1244
49.6907	49.3767
64.6794	64.8987
72.7374	72.8698

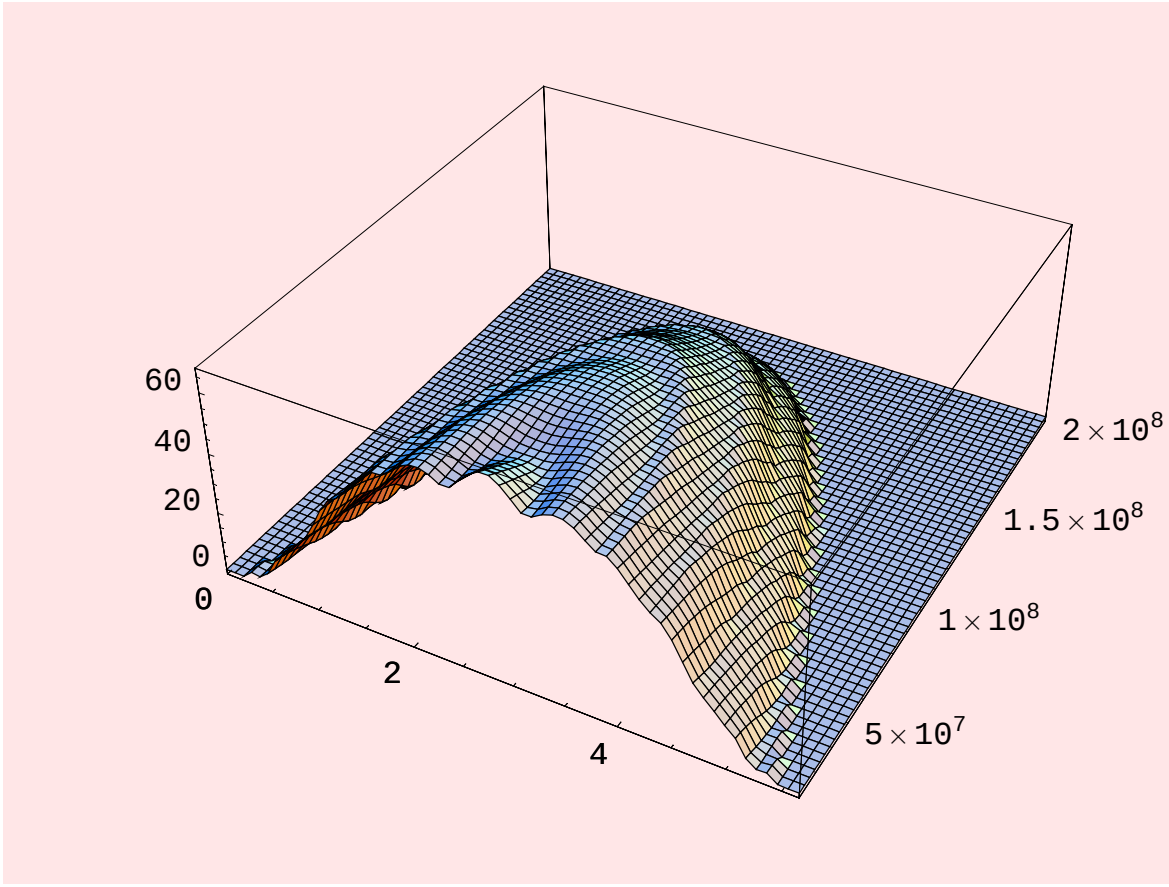
THE TRUNCATION AT THE EDGES EFFECTS THE VALUE OF THE RMS DISTRIBUTION MAKING IT WIDER.

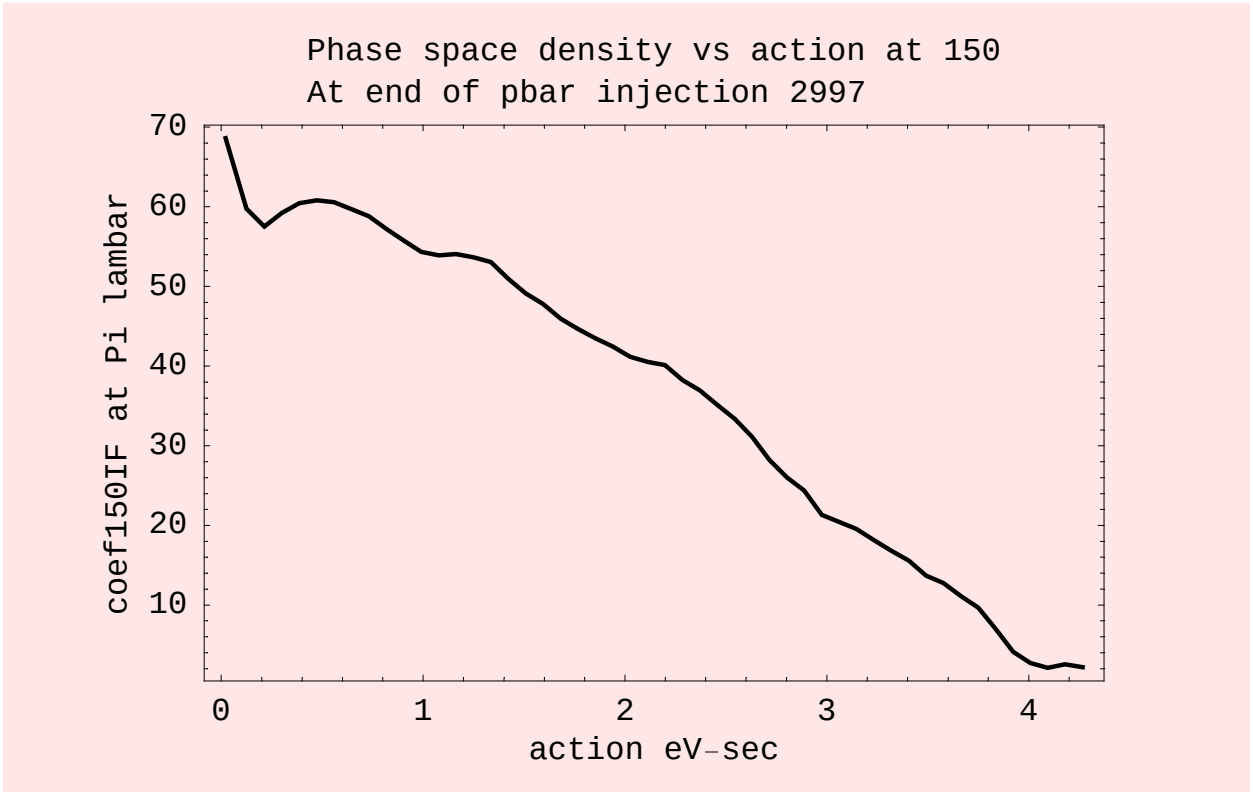
THE PROPER QUESTION THEN IS: DOES THIS DISTRIBUTION CORRESPOND TO WHAT HAPPENS IN THE TEVATRON? THERE IS NOTHING THAT SEEMS TO REQUIRE THAT THE EQUATION FOR ρ TO BE FACTORABLE AND THERE ARE AN INFINITE NUMBER OF OTHER POSSIBILITIES FOR THE FORM OF ρ.

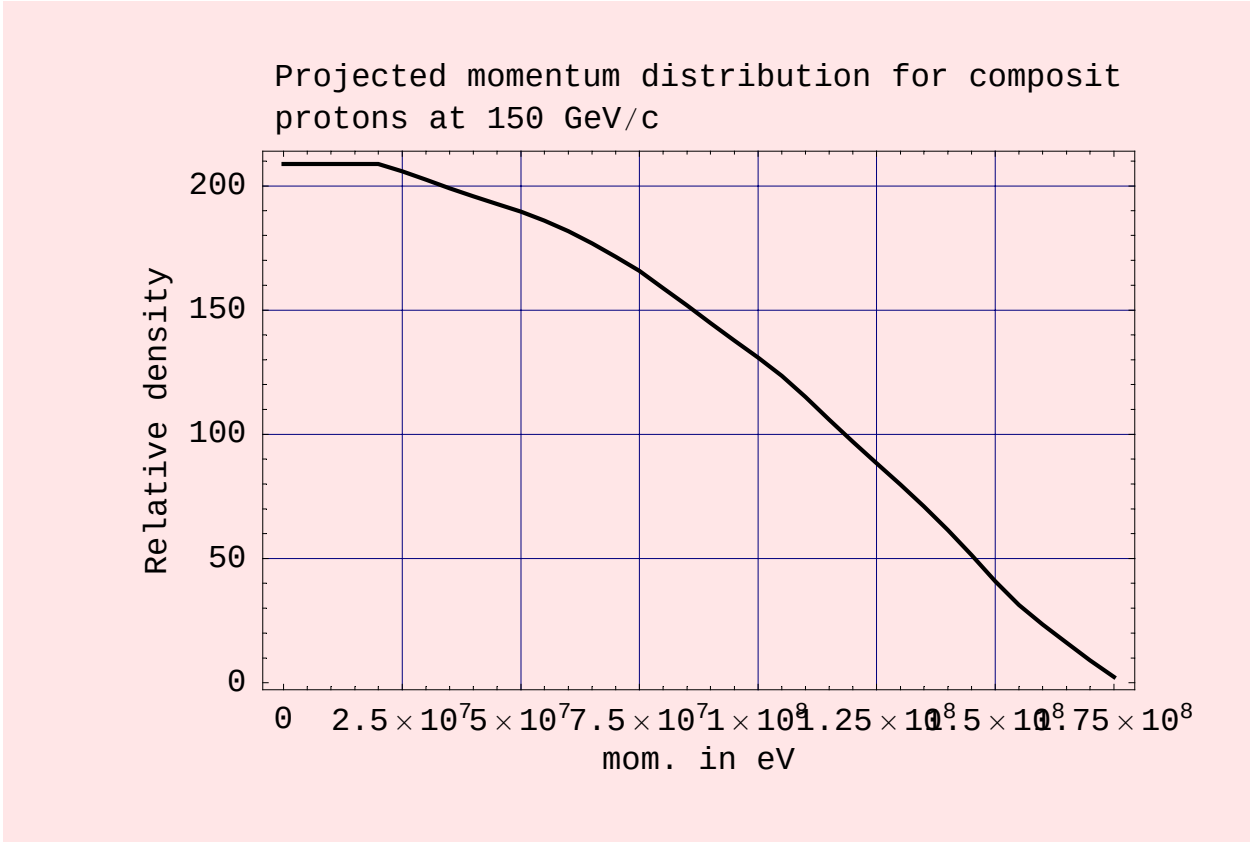
STORE 2997

WE HAVE DATA AT END OF PROTON INJECTION, END OF PBAR INJECTION, AND AT END OF SCRAPING. I WILL ANALYSE THIS DATA USING MY NOTE BEAMS-DOC-548 TO CALCULATE THE PHASE SPACE DENSITY FROM THE TIME DISTRIBUTION. THE CURVE BELOW SHOWS THE SUPERPOSITION OF ALL 36 PROTON PULSES AT THE END OF PBAR INJECTION. THIS ENSEMBLE IS USED FOR TWO REASONS: FIRST IT AVERAGES OVER THE LUMPINESS OF THE PULSES AND GIVES A GOOD PICTURE OF THE TOTAL ACCELERATION PROCESS. SECOND, IT IMPROVES THE SIGNAL TO NOISE RATIO OF THE DATA. STORE 2997, 150 GEV/C.




60
40
20
0
0
2
4
2×10^8
1.5×10^8
1×10^8
5×10^7


Phase space density vs action at 150
At end of pbar injection 2997
coef150IF at Pi lambar
70
60
50
40
30
20
10
0
1
2
3
4
action eV-sec

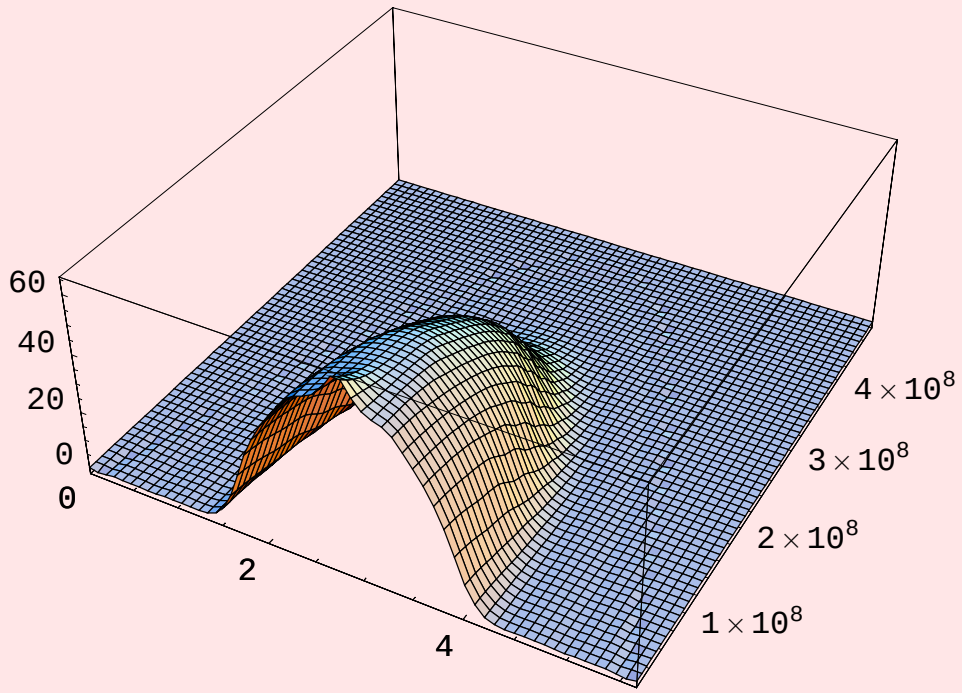


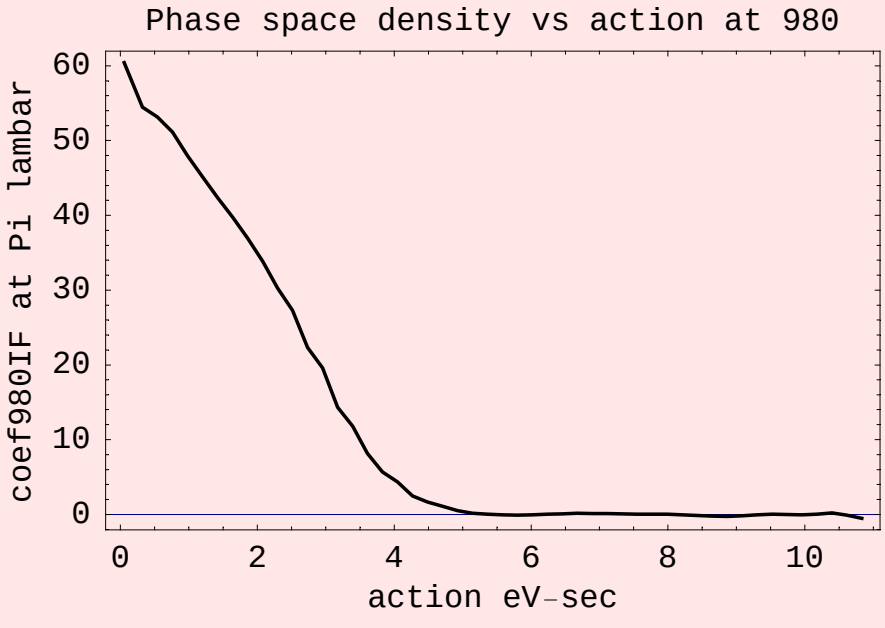
FROM THE ABOVE DISTRIBUTIONS ONE CAN CALCULATE Trms AND Prms. 150 GEV/C PROTONS

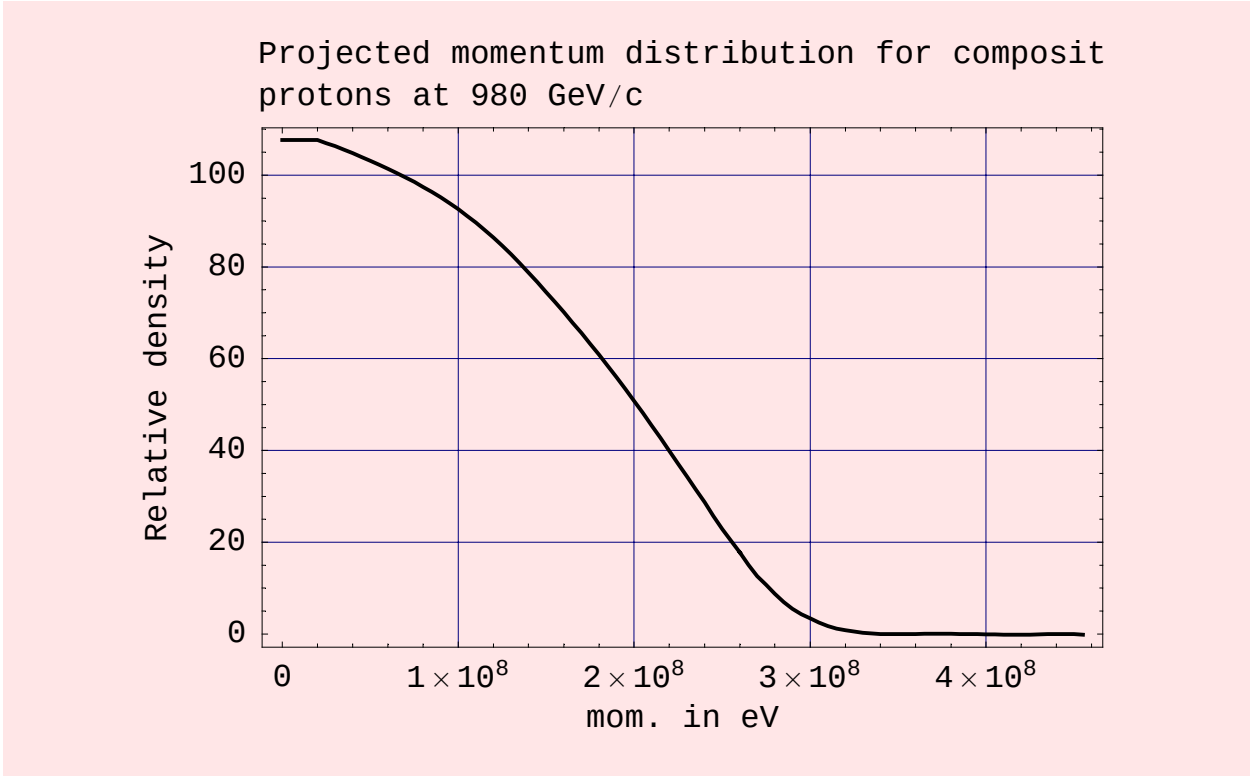
	Trms ns	Prms gev/c	Prms/Trms
CHURCH	**3.14**	**71.8**	**22.8**
2997	**3.14**	**75.2**	**23.4**

THE RATIO IS SUCH THAT CHURCH GIVES A RESULT 1.047 TOO SMALL.

THE MEASURED DISTRIBUTIONS AT 980 GEV/c





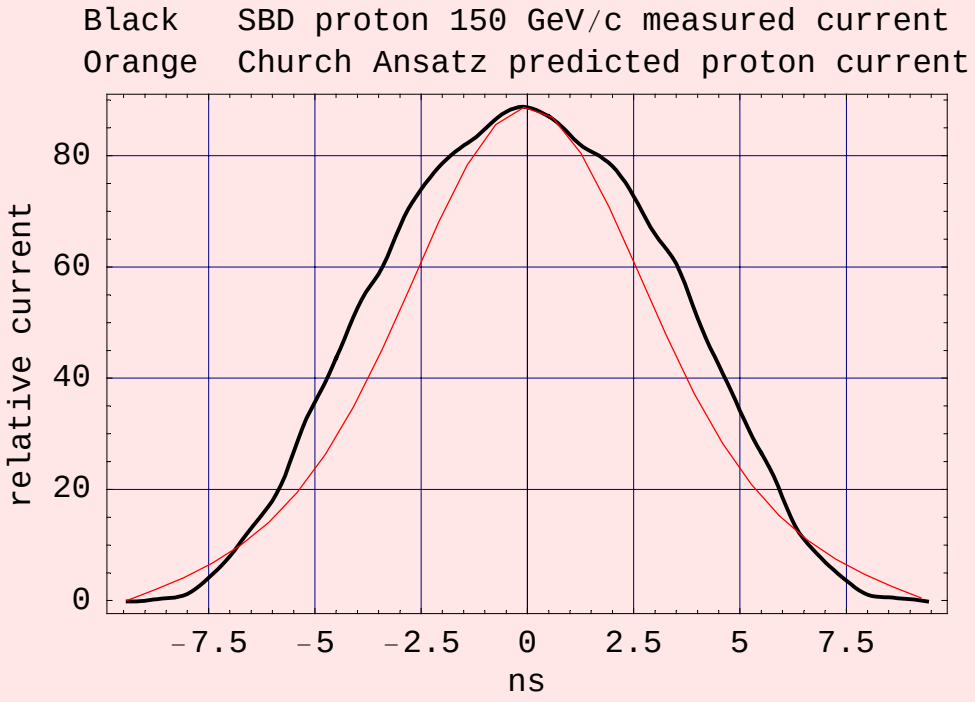


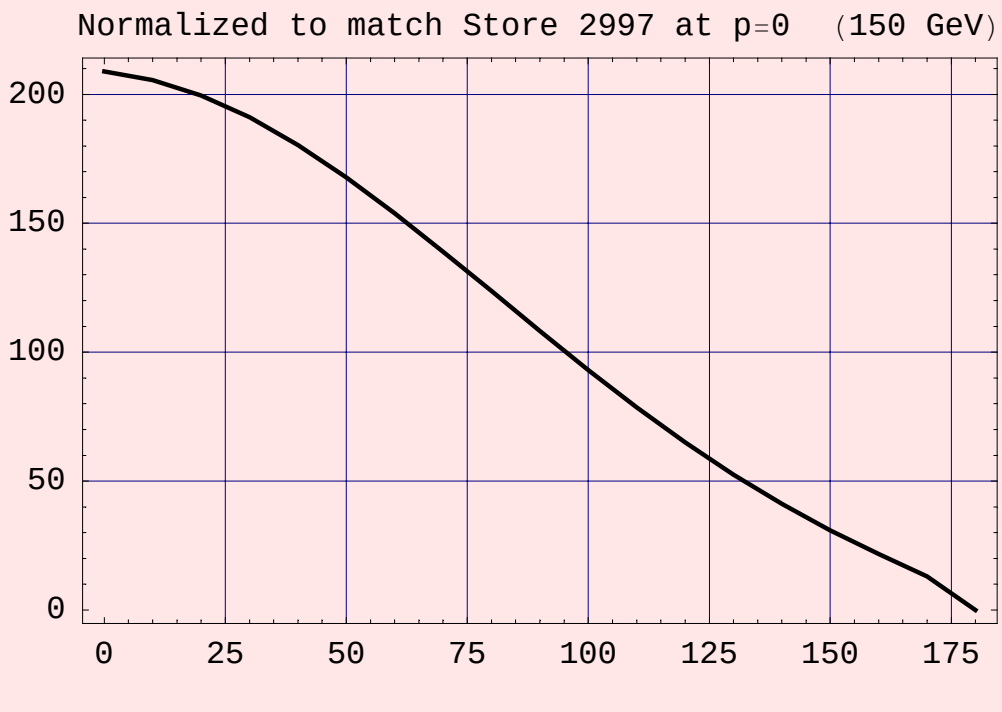
FROM THE ABOVE DISTRIBUTIONS ONE CAN CALCULATE Trms AND Prms. 980 GEV/C PROTONS

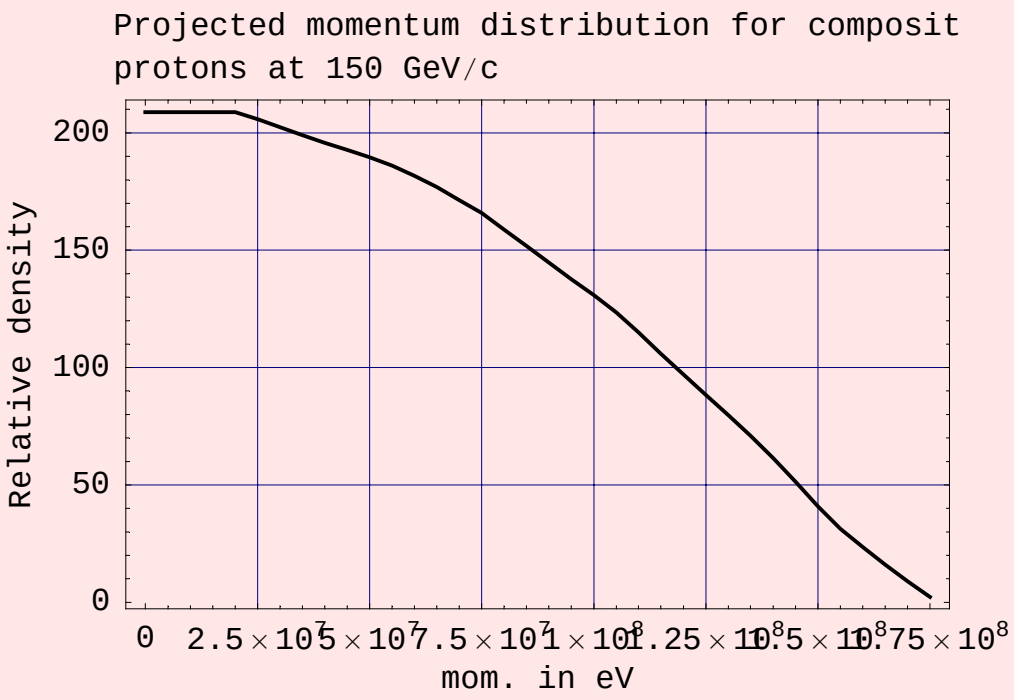
	Trms ns	Prms mev/c	Prms/Trms
CHURCH	**1.85**	**122.9**	**66.5**
2997	**1.85**	**128.2**	**69.4**

THE RATIO IS SUCH THAT CHURCH GIVES A RESULT 1.043 TOO SMALL. SO IS THIS PRETTY GOOD?

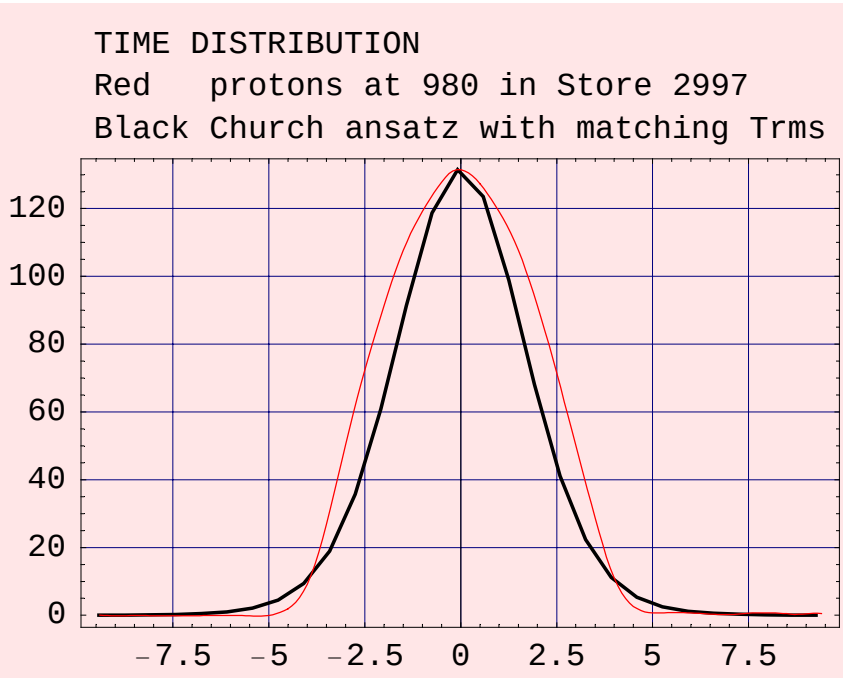
HOW DO THE ACTUALP AND T DISTRIBUTIONS LOOK? THE CURVES BELOW COMPARE THE PREDICTED DISTRIBUTIONS FROM THE CHURCH ANSATZ WITH THE ACTUAL ONES AT 150 GEV/C

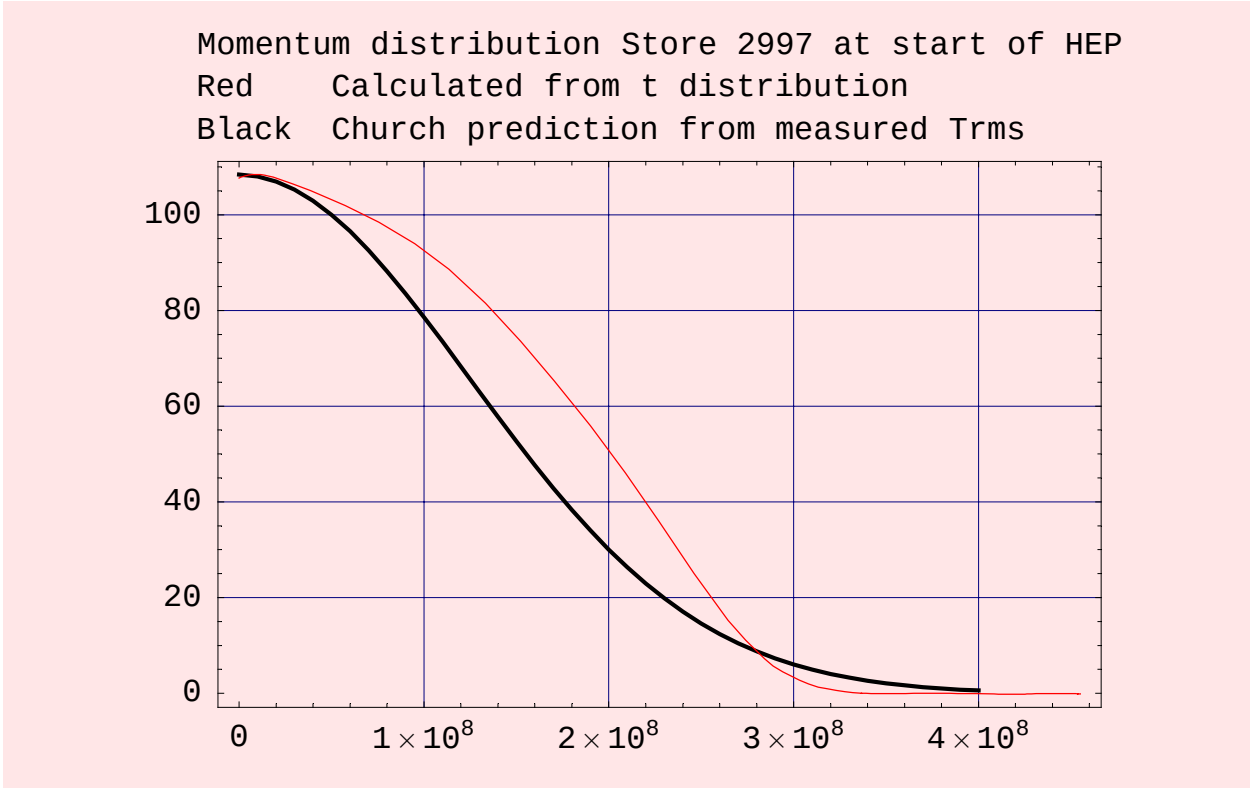






Comparison at 980 GeV/c Start of HEP



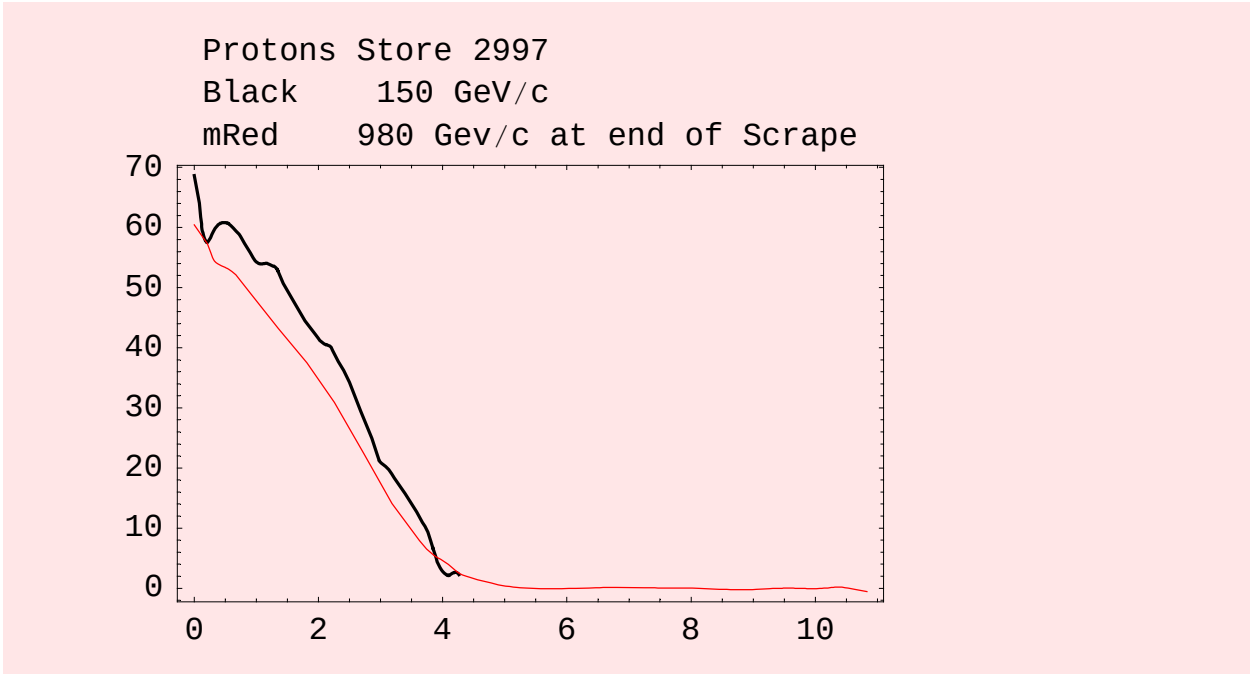


THE ABOVE CHURCH DISTRIBUTIONS ARE BASED ON PICKING

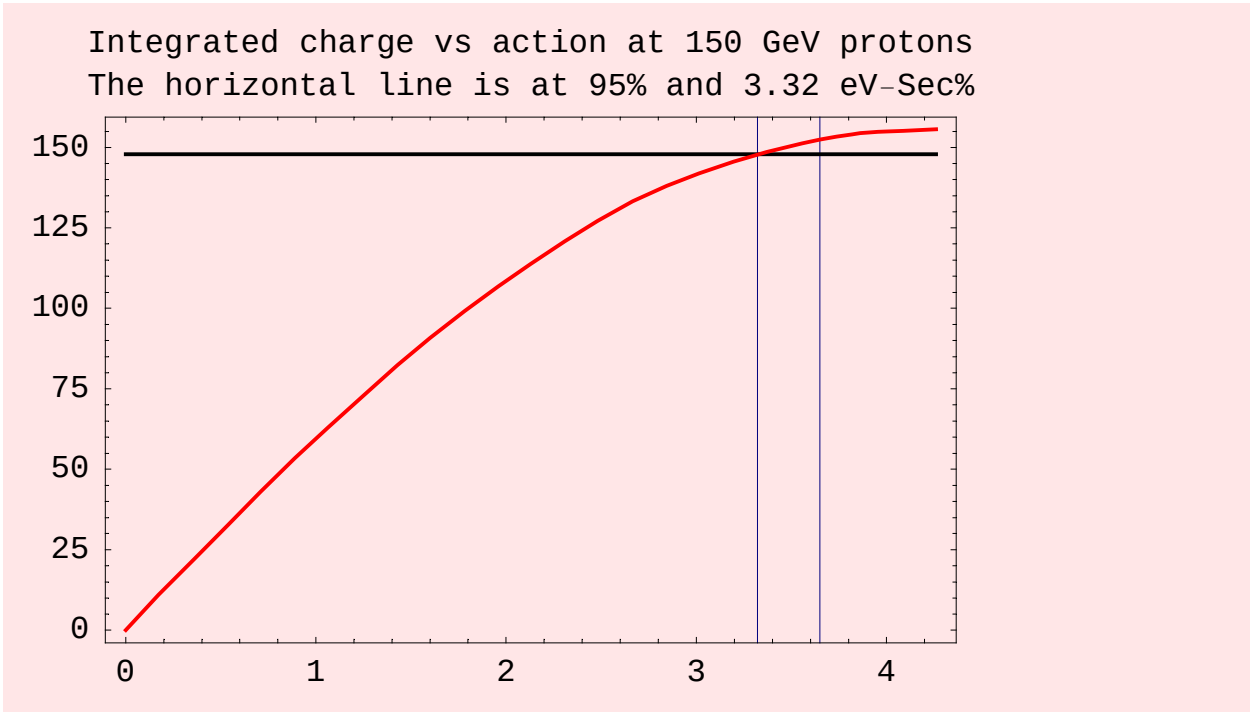
sigE TO GIVE THE OBSERVED Trms FOR THE PROTON BUNCH.

NEXT LOOK AT 95% EMITTANCE. SINCE WE HAVE THE DENSITY DISTRIBUTION, WE CAN INTEGRATE IT TO GET THE 95% eV-sec.

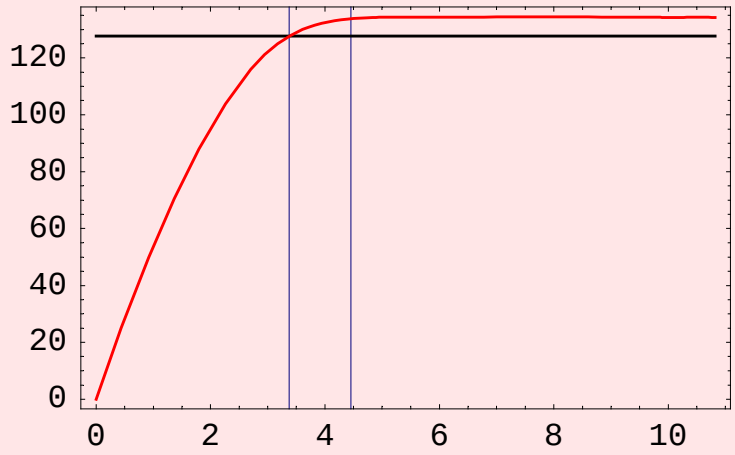
CURVES BELOW SHOW THE DENSITY VS ACTION IN eV-secFOR 150 AND 980 FROM THE UN-FOLDING PROCESS. NOTE THAT THE END-POINT eV-sec IS NEARLY THE SAME FOR 150 AND 980. THERE IS VERY LITTLE DILUTION OF PHASE SPACE.



Intergrating the above curves gives the fraction of particles within a given eV-sec.



Integrated charge vs action at 980 GeV protons
The horizontal line is at 95% and 3.37 eV–Sec%
The second vertical line comes
from the measured Trms and the Church ansatz



THE RED CURVE IS THE INTEGRAL OF THE DENSITY VS ACTION CURVES SHOWN ABOVE. THE HORZONTAL SCALE IS eV-sec. THE HORIZONTAL LINE IS 95% OF THE FINAL VALUE, THE VERTICAL LINE AT THE INTERSECTION IS THE 95% EMITTANCE. THE SECOND VERTICAL LINE IS THE CHURCH ANSATZ.

THE FOLLOWING TABLE SUMMARIZES THE RESULTS. MC STANDS FOR THE CHURCH ANSATZ WHERE sigE IN THE DENSITY HAS BEEN PICKED TO GIVE THE OBSERVED PROTON Trms. THE VALUE USED WAS 86 MeV/c AND 125 MeV/c FOR 150 GeV/C AND 980 GeV/c RESPECTIVELY:

E	Trms	MC Prms	Prms	MC eV–sec	eV–sec
150	3.14	71.8	75.2	3.65	3.32
980	1.85	123	128	4.45	3.37

	Alvin eVsec	Mike eVsec	SBDPES eVsec	Trms SUPERTABLE ns
150	3.32	3.65	8.40 (before ramp case)	3.12
980	3.37	4.45	6.18 (remove halo)	1.81

Thanks to Jean Slaughter for the above table.

SUMMARY

1. The results of this study seem to indicate that there is little emittance blowup during acceleration. This will be investigated by studying more stores and will be reported later.

2. There are many different distributions that have the same rms t and p distributions. The Church ansatz is one

of these that has the special property that the emittance is the product of a function of p and t. However, the actual distributions shown by the beam do not seem to follow this simple form and as a result the predicted 95% emittance is considerably different than that actually present in the beam. It is going to be necessary to install suitable programing to calculate the p distribution from the t distribution and make it available.